Example 5 : Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA; while the other through AC, CD and DA (see Fig. 12.12). Then they cleaned the area enclosed within their lanes. If AB = 9 m, BC = 40 m, CD = 15 m, DA = 28 m and \angle B = 90°, which group cleaned more area and by how much? Find the total area cleaned by the students (neglecting the width of the lanes).

Solution: Since AB = 9 m and BC = 40 m, \angle B = 90°, we have:

$$AC = \sqrt{9^2 + 40^2} \text{ m}$$

$$= \sqrt{81 + 1600} \text{ m}$$

$$= \sqrt{1681} \text{ m} = 41 \text{m}$$
Fig. 12.12

Therefore, the first group has to clean the area of triangle ABC, which is right angled.

Area of
$$\triangle$$
 ABC = $\frac{1}{2}$ × base × height
= $\frac{1}{2}$ × 40 × 9 m² = 180 m²

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The second group has to clean the area of triangle ACD, which is scalene having sides 41 m, 15 m and 28 m.

Here,
$$s = \frac{41 + 15 + 28}{2} \text{ m} = 42 \text{ m}$$
Therefore, area of \triangle ACD = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42(42 - 41)(42 - 15)(42 - 28)} \text{ m}^2$$

$$= \sqrt{42 \times 1 \times 27 \times 14} \text{ m}^2 = 126 \text{ m}^2$$

So first group cleaned 180 m^2 which is $(180 - 126) \text{ m}^2$, i.e., 54 m^2 more than the area cleaned by the second group.

Total area cleaned by all the students = $(180 + 126) \text{ m}^2 = 306 \text{ m}^2$.

Example 6: Sanya has a piece of land which is in the shape of a rhombus (see Fig. 12.13). She wants her one daughter and one son to work on the land and produce different crops. She divided the land in two equal parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get for their crops?

Solution: Let ABCD be the field.

Perimeter = 400 m

So, each side =
$$400 \text{ m} \div 4 = 100 \text{ m}$$
.

i.e.
$$AB = AD = 100 \text{ m}$$
.

Let diagonal BD = 160 m.

Then semi-perimeter s of Δ ABD is given by

$$s = \frac{100 + 100 + 160}{2} \text{ m} = 180 \text{ m}$$

Therefore, area of
$$\triangle$$
 ABD = $\sqrt{180(180 - 100)(180 - 100)(180 - 160)}$

$$= \sqrt{180 \times 80 \times 80 \times 20} \text{ m}^2 = 4800 \text{ m}^2$$

Therefore, each of them will get an area of 4800 m2.

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Alternative method: Draw CE \(\pm \) BD (see Fig. 12.14).

As
$$BD = 160 \text{ m}$$
, we have

$$DE = 160 \text{ m} \div 2 = 80 \text{ m}$$

And,
$$DE^2 + CE^2 = DC^2$$
, which gives

$$CE = \sqrt{DC^2 - DE^2}$$

or,
$$CE = \sqrt{100^2 - 80^2} \text{ m} = 60 \text{ m}$$

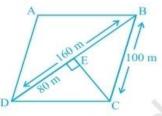


Fig. 12.14

Therefore, area of \triangle BCD = $\frac{1}{2} \times 160 \times 60 \text{ m}^2 = 4800 \text{ m}^2$

EXERCISE 12.2

- A park, in the shape of a quadrilateral ABCD, has ∠ C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?
- 2. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.